Novembertagung
On The History
And Philosophy
Of Mathematics

Conference Programme and Book of Abstracts

33rd Novembertagung on the History and Philosophy of Mathematics

Interactions between History and Philosophy of Mathematics

HOST INSTITUTIONS

University of Rijeka
Faculty of Philosophy
Faculty of Mathematics
Philosophy Dept. – Chair for Logic
Centre for Logic and Decision Theory

VENUE

Faculty of Philosophy, 4th floor, room F-401
Rijeka, 15–16 September 2023

KEYNOTES

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Friday 15th September

9am: Conference opening and welcome addresses by the Deans of the Faculty of Philosophy and the Faculty of Mathematics

9.15am: Luigi Laino, "*Between "Construction" and "Structuralism": Natorp's and Cassirer's Assessment of Metageometry"*

In recent times scholars have devoted their efforts to investigating the role of Cassirer's philosophy of mathematics and geometry in the development of structuralism (Schiemer 2018; Reck 2020). However, less interest has been triggered by Natorp's philosophy of mathematics (except for Mormann 2022). Thus, I aim to newly address Natorp's weight as to the emergence of the non eliminativist structuralist claim that mathematical objects are but "positions in structures" (Heis 2020).

After having outlined similarities concerning the philosophy of arithmetic, I will conversely show that Natorp's and Cassirer's assumptions on geometry are different. My thesis is that only Cassirer's philosophy of geometry is strictly structuralist. This conclusion hinges on what follows. First, Natorp maintains that Euclidean geometry precedes the construction of fundamental structures (unitary segments, spatiotemporal series) and other mathematical objects (such as Minkowski spacetime), although he admits that one can think of multifarious shapes of "coordination" between axioms and experience. Second, I will explain that Natorp seeks sources that tendentially prove his standpoint (such as Mott-Smith), as well as that this may lead him to misjudge some references, such as Klein's *Erlanger Program*, Wellstein and partially Poincaré (Natorp 1910). Quite the opposite, Cassirer appears to be more sensitive to the conceptual implications of the recent philosophy of mathematics, to the point that the concept of "*erzeugende Grundrelation*" is developed in an "analytic" sense in his later works (Cassirer 1929) and impacts his history of geometry, whose pinnacle is the reception of group theory (Cassirer 1910).

In sum, on the one hand, I aim to present Natorp's philosophy of geometry as a direct prosecution of the Kantian constructive method for mathematics. On the other, I will argue that Cassirer gives up intuition (and visualization) in defining the nature of geometrical objects.

Essential bibliography:

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Schiemer, G. (2018). Cassirer and the Structural Turn in Modern Geometry. *Journal for the History of Analytical Philosophy*, 6(3), 182-212.

9.45am: Kate Hindle, "*Placing D'Arcy Thompson in the Historiography of Mathematics*"

D'Arcy Thompson (1860 - 1948), author of *On Growth and Form* (1917), one of the first major biomathematical texts, also held an interest into the study of the history of mathematics. This interest mainly came through in the study of Greek mathematics; he is universally recognised in the literature for his interest in ancient Greece, thanks to his father, a classicist scholar. Thompson also drew on the history of mathematics, and the history of science more generally, in his biomathematical works.

This talk will investigate Thompson's approach to the history of mathematics by creating a framework in which to place him. This will build on pre-existing frameworks, such as Michael Fried's categories of 'mathematician', 'mathematical historian' and 'historian of mathematics' from 'Ways of Relating to the Mathematics of the Past' (2018), and the distinction made between current historians of mathematics and those from the early 20th century by Reviel Netz in *A New History of Greek Mathematics*. This description of 1920s mathematics history can be used to put Thompson into the context of the historiography of (particularly ancient) mathematics of his time, which will allow for assessment of Thompson against his peers.

10.15am: Coffee Break

10.30am: Hywel Griffiths, "*The effectiveness of mathematics*"

The real foundation of mathematics, as opposed to its logical foundation in set theory or type theory, is our perception, action, and capacity for language. These have combined over millennia to develop a symbolic technology for solving problems in the physical world. The effectiveness of mathematics in the natural sciences, rather than being a puzzle as Wigner [2] suggested, is its core feature, explaining its origin and development.

Perception and action are fundamental functions of animal nervous systems, with the posterior half of our cortex devoted to perception and the frontal half to action [1]. In the human case these combine with our capacity for language and symbolic representation to allow procedures such as counting and measuring, which in turn underlie mathematics and physics.

The role of our perception as a foundation for mathematics is indicated by Euclid's first proposition, to construct an equilateral triangle (Figure 1). The proof requires that the two circles meet at a point, but this isn't implied by the explicitly stated axioms. Instead, this is supplied by our visual system when looking at the corresponding diagram. Our visual system therefore supplies a logical axiom in the foundation of geometry.

I will argue that a procedural core, based on rule-following, is the basis for the autonomy and objectivity of mathematics, and therefore we don't require either physical nor platonic referents to mathematical terms in order to account for this objectivity.

However, the foregoing leaves open a more specific question than Wigner's about the applicability of mathematics: why do some mathematically significant concepts turn out to be physically significant? An example is simply-connected groups, which are mathematically significant among groups, and the physical quantity of spin. I will outline the case of the group SL2C, the simply-connected double cover of the Lorentz group, which is required to derive spin.

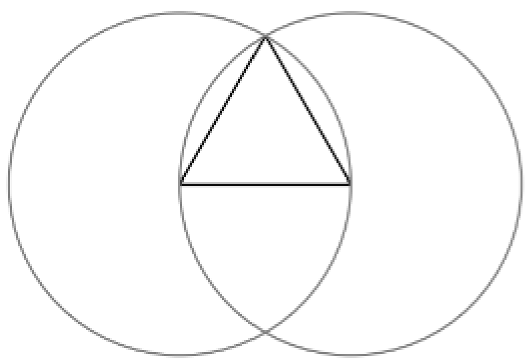

Figure 1: Theorem 1 from Book 1 of Euclid's Elements

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[1] Joaquín Fuster. *The Prefrontal Cortex*. Academic Press, London, fifth edition, 2015.

[2] Eugene Wigner. The unreasonable effectiveness of mathematics in the natural sciences. *Communications in Pure and Applied Mathematics*, 13:1–14, 1960.

11am: Zachary Stanley, *"Formalisation and the Definition of the Function Concept*"

The presentation of fractal curves in 1872 by Weierstrass precipitated great breakthroughs in the study of functions, in particular the class of "nowhere differentiable" functions. Yet, a widely reproduced proof by Ampère from 1806 — endorsed or at least condoned by many contemporary mathematicians, including Gauss, Cauchy, Duhamel, Bertrand and Gilbert[1] — was believed to have established that all continuous functions are differentiable. While some have emphasised the implicit premises of Ampère's proof[2] simply conditionalise its conclusion to continuous functions or otherwise make it illegitimate, others[3] have been more concerned with why it was convincing, whether it was knowledge-imparting, and what the proof demonstrates about the nature of mathematical reason- ing; for instance, whether Weierstrass's presentation undermines or *revises* Ampère's original result.

I propose to chart the history of the function concept with re- spect to Ampère's "proof," and conjecture that fractal curves constitute a *synchronic change* to the function concept. I offer both that Fourier's (1822) studies of functions were highly instrumental in this revision, and that Weierstrass's results were more initially contentious than is often supposed. I then contrast how synchronic revisions such as this case study might be understood with respect to the frameworks of *domain extention* (following Manders 1989), quasi-empiricism (Lakatos 1976), and distinct mathematical formalisations. I claim principally that the function concept displays open texture that was closed gradually with formal redefinition following a series of key developments.

In detailing a key interaction between the history and philosophy of mathematics, this talk will further our understanding of the mathematical practices and relate them to cogent philosophical ac- counts of mathematical truth and epistemology. It will elucidate the role of informal and open-textured mathematical concepts in delimiting which proofs are acceptable and how they are interpreted by mathematicians.

1. (Singh 1935)
2. (Jordain 1913)
3. (Kline 1990, p.955)

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11.30am: Keynote Speaker Georg Schiemer, "*How Geometry Became Structural*"

Structuralism in the philosophy of mathematics is, roughly put, the view that mathematical theories study abstract structures or the structural properties of their subject fields. The position is strongly rooted in modern mathematical practice. In fact, one can understand structuralism as an attempt to come to terms philosophically with a number of wide-ranging methodological transformations in 19th and early 20th century mathematics, related to the rise of modern geometry, number theory, and abstract algebra. The present talk will focus on the geometrical roots of structuralism. Specifically, we will survey some of the key conceptual changes in geometry between 1860 and 1910 that eventually led to a "*structural turn*" in the field. This includes (i) the gradual implementation of model-theoretic techniques in geometrical reasoning, for instance, the focus on duality and transfer principles in projective geometry; (ii) the unification of geometrical theories by algebraic methods, specifically, by the use of transformation groups in Felix Klein's *Erlangen Program*; and (iii) the successive consolidation of formal axiomatics in work by Hilbert and others.

12.30pm: Lunch Break

1.30pm: Benjamin Wilck, "*Are Definitions Boundaries of Concepts Only Metaphorically? A Reply to Frege*" (Zoom)

In this paper, I tackle Frege's claim that all scientific definitions are sharp boundaries of concepts only metaphorically. In particular, I argue that many geometrical definitions are literally boundaries. Frege conceives of scientific definitions as sharp boundaries, namely, boundaries of concepts (*Die Grundlagen der Arithmetik*, 1884, §§ 26, 88; *Grundgesetze der Arithmetik II*, 1903, §§ 56–65). Though he takes boundary-talk about definitions and concepts to be but metaphorical (1903, § 56). That is to say, Frege claims that all definitions are boundaries only in a figurative, non-literal sense. Against the view that all definitions are boundaries of their concepts merely metaphorically, I argue that there are scientific definitions that are literally sharp boundaries of concepts, namely, geometrical definitions.

Already Euclid's geometry shows that the terms in a definition may be literally boundaries. Euclid defines, for instance, the triangle as *a rectilinear plane figure contained by three straight lines* (*Elements* I.def.19ii). Thus, a triangle is not only literally bounded by straight lines, but also defined by the terms *straight* and *line*.

All geometrical figures are defined by reference to their literal boundaries. This conceptual interdependence of *boundary* and *figure* is echoed also in Euclid's definitions of these two terms: "A boundary [*horos*] is that which is the limit [*peras*] of something. A figure [*schêma*] is that which is contained by a certain boundary [*horos*] or certain boundaries [*horoi*]" (*Elements* I.def.13–14).

More generally, then, the boundaries by which a geometrical figure is contained are contained in its definition. This overturns Frege's claim that all scientific definitions are sharp boundaries of concepts merely metaphorically.

2pm: Frederike Lieven, "*Intellectual background for the « New Math » reform in France and Germany*"

In the 1960s, many countries undertook profound reforms in the teaching of mathematics, introducing new content and perspectives, such as set theory and algebraic structures. The most common explanation for those reforms is the need to train more engineers and technicians in order to ensure continued scientific and economic progress, especially after the Sputnik crisis in 1957. However, the mathematical content emphasised by the reformers has no obvious technical applications.

The focus on modern mathematics at all levels of schooling can be linked to underlying philosophical ideas, that were common in the 1960s. An analysis of the discourse of the reform's leaders shows an intellectual climate characterised by the belief that modern mathematical thinking is necessary to understand the present and, more importantly, the future world. According to this vision, teaching abstract algebra to schoolchildren becomes an act of emancipation.

In this presentation, I show how various key players in the reform process such as mathematicians, mathematics teachers, politicians... expressed those ideas and how they were articulated in the public discourse about science, as seen in the media. In particular, I examine whether there were any effective arguments in support of the idea that modern mathematics are necessary to live in a rapidly changing world. I discuss how the two main goals of the reform emancipation of mankind and economic growth can be complementary or contradictory. Finally, I assess the emphasis placed by the reform on teaching children to think for themselves with regards to its political and generational consequences.

In the light of these reactions, the abandonment of the reform in the 1970s also indicates that the 1960s vision of mathematics in society is outdated.

2.30pm: Cecilia Neve Jiménez, *"A Historic Review of Representations of the « Mediant », Farey sequences and Related Topics"*

The « *mediant* », namely, the operation that yields $\frac{a}{b} \oplus \frac{c}{d} = \frac{a+c}{b+d}$, constitutes a binding element among diverse, yet closely related subjects that span across what in the present day are conceived as different branches of mathematics. A basic element in continued fractions and rational approximations, the mediant also plays a fundamental role in Farey series and analogous sequences, as well as in the construction of rational numbers depicted by the Stern-Brocot tree; in addition, it naturally appears in the modular group $\mathrm{PSL}(2, \mathbb{Z})$.

Although it was mainly during the 19th and particularly the 20th century that those subjects and their interactions proliferated, the mediant can be traced back to Chuquet's 1484 manuscript *"Le Triparty"*. From this text onward, the diverse graphic representations of the mediant and related elements present a rich opportunity to contrast the notational and diagrammatic resources that were employed, taking into account their historical context. These representations enable us to question what is intended to be depicted and the motivations behind those depictions.

In this talk, we will make a brief historical review of some of these graphical representations, pointing out some of their features. We will explore the first descriptions and notational aids of Chuquet's method, the depiction of the sequences that arose from Lagrange's works on Number Theory, the tables from early XIX century calculators, the Brocot sequences that emerged from his work as a clockmaker, as well as the transition to more "formal objects" such as Hurwitz's Farey Polygon and the Stern-Brocot tree.

3pm: Coffee Break

3.30pm: Kaveh Boveiri, *"Hegel and Mathematics: Towards the Resolution of a Dilemma"*

The relation of Hegel's thought to mathematics is ambiguous. On the one hand, he repeatedly expresses an explicit distrust of mathematics. This distrust reveals itself in several aspects: mathematical argumentation remains outside *Sache selbst*; it solidifies reality; it is limited to form without being able to penetrate to content; the essence of proof of such argumentation does not possess the nature of being a moment of its result, and one can add other elements to it. If this is the case, apparently, we do not have to witness a defense of mathematics in his works. However, we note that he tells us, in the first volume of *The Science of Logic* and in his discussion of the differential calculus, that if the binomial expansion of the differential polynomial is well understood, namely, $dx^n = nx^{n-1}\,dx$, the ratio dy/dx can be seen as a qualitative and dynamic relationship between different quantities.

Moreover, such a ratio, by presenting itself as the constituent of motion, provides us with the true infinity. Thus, despite his distrust of mathematics, according to him the true infinite can be found in mathematics. This paper aims at evaluating the following hypothesis: mathematical argumentation (including that of geometry) can represent the development of Sache selbst under the condition of the possibility to overcome the limits [*Grenzen*], but also the obstacles [*Schranken*] of mathematics. Such a reading is suspicious of the idea of the dynamization of reality, according to which movement is integrated or injected into static logic. Instead, it proposes the recognition of the overcoming dynamism inherent in Sache selbst, a dynamism also recognizable by such a mathematical argumentation.

4pm: Keynote Speaker Zvonimir Šikić, *"Are there mathematical concepts that are real?"*

According to [D], C. F. Gauss said: If e iπ = -1 was not immediately apparent to a student upon being told it, that student would never become a first-class mathematician. We will explore the arguments that support Gauss's claim in order to prove that there are no mathematical concepts that are real in Steiner's sense.

We conform to the position that concept exists if it satisfies the W. O. Quine's condition: Fs exist if ∃xFx is a theorem of a true theory; cf. [Q]. But M. Steiner claims in [Sr] that it is possible for Fs to satisfy this condition without being real. His inspiration is P. Bridgman's definition of physical reality: Something is physically real if it is connected with physical phenomena independent of those phenomena which entered its definition; cf. [B] p.56.

There is something profoundly right in the idea that the real is that which has properties transcending those which enter its definition and Steiner's aim is to show that mathematical entities can occasionally be said to be real in exactly the same sense.

Quine's condition is applicable to the existence of mathematical entities: scientific theories are committed to the existence of mathematical entities, and since we regard some of them as true, we must regard mathematical entities as existent. However, according to Steiner, this is not an argument for the reality of mathematical entities.

To demonstrate the reality of an entity in the natural sciences one typically shows that the entity is indispensable in explaining some new phenomenon. In this way the entity acquires new and independent descriptions. Steiner applies the same idea in mathematics.

For example, π is real because we have at least two independent descriptions for π. Geometric, π = C/2r and analytic, π = ln (-1)/i. In the first case π is derived from the formula for the circumference of a circle C with radius r. In the second case π is derived from the special case of Euler's formula, e πi = -1.

We know by deductive proof that the descriptions are coreferential (unlike the situation in the physical sciences where this is demonstrated empirically). But then, how can probably coreferential descriptions be regarded as independent?

Steiner’s answer is to distinguish between two kinds of proof of coreference in mathematics: those which are nonexplanatory and merely demonstrate the coreference, and those which explain it. Descriptions are independent if the proofs of their coreferentiality are nonexplanatory.

We show that the "independence of the descriptions of two mathematical entities" is not additionally explained by the "absence of explanatory proofs of their coreference", so we will stick with “independence” as a less vague criterion.

After a detailed analysis of the "reality status" of π, in the previously described context, we conclude that π is not real in Steiner's sense. As a matter of fact, it is difficult to prove for any mathematical concept that it is real in Steiner's sense. Namely, it is not enough to formulate two descriptions of a concept and find a proof of their coreference which keeps the descriptions independent. It should be proved that all proofs of their coreference are such.

But mathematical theories are deeply connected and in the entire history of mathematics, mathematicians are constantly striving to discover these connections. For example, it is typical for mathematicians to persistently search for new proofs of old theorems in order to discover these intertheoretical dependencies.

Hence, our hypothesis is that no mathematical concept is real in Steiner’s sense.

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Saturday 16th September

9am: Paul-Emmanuel Timotei, "Halphen *more geometric* than Noether"

For the French translation of George Salmon's *A Treatise on the Higher Plane Curves* (translation by O. Chemin published in 1884), Georges-Henri Halphen (1844–1889), composed an appendix titled "Etude sur les points singuliers des courbes algébriques planes [Study of the singular points of plane algebraic curves]." The third part of this appendix, deals with questions regarding the reduction of singularities. Halphen presents a method that was new at the time, and that seems to have been forgotten today. This method allows him to obtain a result for the reduction of singularities of plane curves. This result, Halphen claims, can be generalized easily and allows him to obtain the following theorem, which Max Noether had published a few years earlier:

> To any algebraic curve there corresponds point by point another curve, in such a way that all singular points of the former curve correspond to simple points of the latter, which itself has no other singular points than ordinary double points.

Following the statement of the theorem, Halphen clarifies this:

> The analysis developed in Sections 58 and 59 [where the method he followed to reduce singularities is described] leads me to give M. Nöther's theorem a more geometric form.

What does Halphen mean with words *a more geometric form*? My talk will offer an interpretation. Moreover, I will address the following question:

What allows Halphen to judge his approach to Noether's theorem *more geometric*?

To explore this issue, I will present Halphen's method in the appendix intuitively; I will then examine the way in which the theorem appears in Noether's articles; I will discuss Halphen's training as well as the interpretation of these different methods by their readers.

9.30am: Mireia Martinez i Sellarès, *"The curves are no longer similar": On the Beginnings of Geometrical Affinity"*

The notion of affinity between curves was introduced by Leonhard Euler in his Introduction to the Analysis of Infinity (1748), and towards the end of the 18th century the term had entered other mathematical textbooks such as the Mathematische Anfangsgründe by Abraham Kästner. A few decades later, August Ferdinand Möbius devoted an entire chapter to it in his Barycentric Calculus (1827) where he, interestingly, disagreed with some of Euler's remarks about affinity in the context of a change of coordinates.

In this talk, we explore the emergence and adoption of the notion of affinity in mathematics in the late 18th century and early 19th century. Firstly, we present Euler's definition and examine how he used it in his own work. Secondly, we compare Euler and Möbius' notions of affinity and analyze the significance of their differences: from our present-day perspective, Möbius' definition is closer to the modern one, and his disagreement with Euler's statements can be interpreted as a valid critique of Euler's definition being dependent on a particular coordinate system.

Thus, tracing the beginnings of affinity (and with them, of affine geometry) constitutes an interesting case study in the coinage of new terms in mathematics and provides evidence on the implicit and explicit choices that mathematicians make when coming up with new definitions. In a broader sense, the discussion between actors on what affinity is or ought to be offers valuable historical insight into the transition from classical computational to modern structural conceptions of analytic geometry.

10am: Coffee Break

10.30am: Ravi Chakraborty, "*From Boole to Cassirer: Algebra as the law of thought and perception*" (Zoom)

The historical development of algebra, like the development of mathematics itself, provides resources for philosophers to think about thought and perception in general. My particular interest is in the reception of algebra by two philosophers: George Boole and Ernst Cassirer. Boole believed that an algebra of logic could furnish the laws of thought. In this framework, the algebra seems to enact 'mental operations' such as selection, for example. While analyzing this foundational impulse that accords a certain primacy to algebra, we will trace the shift that happens in the reception of algebra with the emergence of group-theory.

This influence of group theory is most strongly felt in the thought of Ernst Cassirer. While I will show how both Boole and Cassirer exploit the power of algebraic notions, they approach the universality of algebra from different directions. For instance, Boole suggests that algebra should speed up logical calculations. Cassirer emphasizes the role of invariance and transformation in perception.

The nagging question I wish to address is: does mathematics help to speed up thought through new possibilities of symbolic manipulation, or do new advancements in mathematics reveal the structure of thought better? While this issue has been somewhat explored in the context of Kant's interest in geometry: the rough contours of the proposition being that Kant considers mathematics as a key example of synthetic a priori knowledge. The subsequent question being how do new developments in mathematics cast a shadow on Kant's thought? My paper will aim to show that this conversation about the influence of the historical development of mathematics is more richly demonstrated as we move from Boole to Cassirer and may help us understand why and how algebra came to play a role in not just unifying mathematics but also the laws of thought and perception in general.

11am: Marija Šegan-Radonjić, "*Mihailo Petrović and the Mathematical Institute of the Serbian Academy of Sciences*"

Mihailo Petrović, the Serbian mathematician and the founder of the Serbian School of Mathematics, was one of the first advocates of the idea to set up a specialized institution for Mathematical Sciences in Serbia at the end of the 19th and the beginning of the 20th century. Although he was inspired by the French School of Mathematics, he was aware that due to a lack of financing and teaching staff, it was not possible to establish such an institution immediately. The first more concrete step towards setting up a separate institute was made in mid-1938, immediately after Petrović retired as a university professor. To commemorate the occasion, his colleagues proposed establishing two independent institutes "Institute for Theoretical Mathematics: Dr. Mihailo Petrović" and "Institute for Applied Mathematics." Although the proposal was adopted by the University of Belgrade, unfortunately, it was not implemented due to the outbreak of World War II. Nevertheless, it served as the inspiration to set up a specialized institution that continued Petrović's work on the development and dissemination of mathematical knowledge – the Mathematical Institute of the Serbian Academy of Sciences, founded after World War II, in 1946. Sadly, Petrović did not live to see the establishment of this institution, but he left a busting "hive of scientific work."

This paper looks into Petrović's role in establishing the Mathematical Institute of the Serbian Academy of Sciences, as well as the intention of the Institute's founders to continue his mission in post-World War II Yugoslavia. It analyzes the first years of the Institute's work and concludes that the Mathematical Institute embraced Petrović's legacy and contributed to further development of Mathematical Sciences in Serbia and former Yugoslavia.

11.30am: Keynote Speaker Silvia De Toffoli, *"Disagreement in Mathematics: Why It Matters?"* (Joint work with Claudio Fontanari)

If there is an area of discourse in which disagreement is virtually absent, it is mathematics. After all, mathematicians justify their claims with deductive proofs: arguments that entail their conclusions. But is mathematics really exceptional in this respect? Looking at the history and practice of mathematics, we soon realize that it is not. First, deductive arguments must start somewhere. How should we choose the starting points (i.e., the axioms)? Second, mathematicians, like the rest of us, are fallible. Their ability to recognize whether a putative proof is correct is not infallible. In most cases, disagreement over the correctness of a putative proof is, however, evanescent. Once an error is spotted and communicated, the disagreement disappears. But this is not always the case. Sometimes it is recalcitrant; that is, it persists over time. In order to zoom in on this type of disagreement and explain its very possibility, we focus on a single case study: a decades-long (1921-1949) controversy between Federigo Enriques and Francesco Severi, two prominent exponents of the Italian school of algebraic geometry. We suggest that the instability of the mathematical community to which they belonged can be explained by the gap between an abstract criterion of rigor and local criteria of acceptability. It is this instability that made the existence of recalcitrant disagreement over putative proofs possible. We do not condemn speculative mathematics but rather its pretense of being rigorous mathematics. In this respect, we show that the overly self-confident Severi and the more intuitive, visionary Enriques had a completely different attitude.

12.30pm: Lunch Break

1.30pm: Benjamin Zayton, *"Qualifying the Received View on Urelements in Set Theory"*

In the philosophy of set theory, the received view on urelements is that urelements are necessary to account for the applicability of set theory outside of mathematics, but dispensable for theoretical purposes. In this essay, both components of this view will be systematically examined. As groundwork, there first is a brief recounting of the role of urelements in 20th-century mathematics, showing that the received view goes back to Zermelo.

Thereafter, three arguments for the theoretical dispensability of urelements are discussed.The first, Reinhardt's structuralist argument, argues that all structural questions about (impure) sets can be reduced to questions about pure ordinals. To resist this, it can be pointed out that this says little about the structural features of models of set theory, and that the representational role of set theory requires consideration of non-structural features.The second argument uses bi-interpretability results between set theory and set theory with urelements to argue that the latter is dispensable. However, strong equivalence results, obtained by Löwe, require problematic assumptions on the size of the class of urelements, while more general results obtained by Hamkins and Yao can be attacked on the grounds of requiring the addition of parameters to the set-theoretic language. The quasi-empirical third argument is based on the hitherto expressive adequacy of set theory without urelements, and can be resisted by noticing that set theory with urelements has other Maddian foundational virtues.

Finally, the last section of the paper shows that on popular accounts of mathematical representation absence of urelements is either superfluous for structuralist reasons or a harmless idealization. In light of this, the best way for the defender of the received view seems to be the adoption of strong metaphysical claims, such as a Siderian view on the book of the world.

2pm: Hala Khassiba, ***"From pure mathematics to applied mathematics: emergence of a new discipline at University of Nancy after the Second World War"*** (Zoom)

After the Second World War, Nancy became one of the first French provinces starting studies in what would later be called computer-science. Regarding to a tradition of pure mathematics established by the Bourbaki at the Faculty of Science in Nancy, Jean Legras, settled a postgraduate course in Numerical Analysis in the department of mathematics and made it possible to obtain and use a various electronic computer such as IBM 604, IBM 650, and others.

How to explain the emergence, from the 1950s, of a new discipline? And how to interpret the fact that a small community of mathematicians turned away from pure mathematics, at the time of Bourbaki, to initiate new research on the first programming languages?

2.30pm: Coffee Break

3pm: Daniel Usma Gomez, ***"Aquinas and Benacerraf: some remarks on the topicality of medieval philosophy of mathematics"*** (Zoom)

In this talk, my aim will be to propose some remarks on one of the best-known questions in contemporary philosophy of mathematics: Benacerraf's dilemma, which is often seen as a major challenge for mathematical realism.

According to the many formulations of the dilemma, the belief that mathematics treats *abstract* objects would be in conflict with a widely accepted *causal* theory of knowledge, for *abstractness* would consist in a *causal inertia*. Realism would be then committed with an unworthy epistemology. In virtue of this, many have denied the existence of abstract objects, refused any metaphysical commitment, and opted, at best, for a fictionalist or a pragmatic interpretation of mathematics, with a better epistemology. This partly shows that philosophers of mathematics have, in the later 20th century, somehow faced a kind of Shakespearean situation: "to be or not to be *realist?* That's the question".

I would like to suggest that such a binary attitude results from a particular understanding of *abstractness*, which is certainly not the only one. In light of a Thomistic-Aristotelian account for *abstraction,* I will try to show that one may to be moderately *realist* without contravening to a widely accepted causal theory of knowledge. I will draw on some conceptual elements of Thomas Aquinas' views on mathematics which are in no conflict with views on abstraction in today's mathematical practice in order to show that we s8ll could be realist without turning our aEen8on from concrete mathematical practice.

3.30pm: Keynote Speaker Karine Chemla, *“Mathematical collectives according to observers and actors: The historiography of numeration systems and arithmetic”*

The historiography of numeration systems and arithmetic has been for its greatest part organized according to nations and linguistic groups. This holds true, for Geneviève Guitel in her Histoire Comparée des Numérations Ecrites (1975), despite the fact that the thesis for which she argues aims at highlighting key general principles according to which all numeration systems can be described and classified. One might argue that this way of approaching numeration systems is justified inasmuch as actors themselves have thought in these terms. Suffice it to mention that medieval Arabic treatises of arithmetic often refer to “Indian reckoning”. I will first examine how we can interpret such expressions that actors are using. Secondly, I will show that, in contrast with observers’ approach to number systems, such as Guitel’s, this is not the way in which, in his Kitab al-Fusul fi al-Hisab al-Hindi (Saidan 1978), the 10th century practitioner of mathematics, al-Uqlidisi perceived the divide between different numeration systems and their collectives of users. In a third part, I return to

the example of Guitel’s historiography of number (1975), to highlight some widespread tacit hypotheses that have more broadly maintained the view that national or linguistic groups constituted the relevant frames of analysis in the historiography of numeration systems.

Lastly, I will show how jettisoning these hypotheses opens up new perspectives for discussing the nature of number and numerals. From his account, I will suggest that we can derive a research program about numeration systems and arithmetic that should allow us to highlight the different cultures of computation that coexisted in ancient societies, as well probably as in modern ones.

References

Guitel, Geneviève, 1975. Histoire Comparée des Numérations Ecrites (Paris: Flammarion, 1975).

Saidan, Ahmad S., 1978. The Arithmetic of al-Uqlidisi. The Story of Hindu-Arabic Arithmetic as told in Kitab al-Fusul fi al-Hisab al-Hindi, by Abu al-Hasan Ahmad ibn Ibrahim al-Uqlidisi, written in Damascus in the year 341 (A.D. 952/3) (Dordrecht: D. Reidel, 1978).

4.30pm: Closing of the Conference and concluding discussion about the organisation of the 34th Novembertagung (2024)

8pm: Conference dinner @ Konoba “Tarsa”